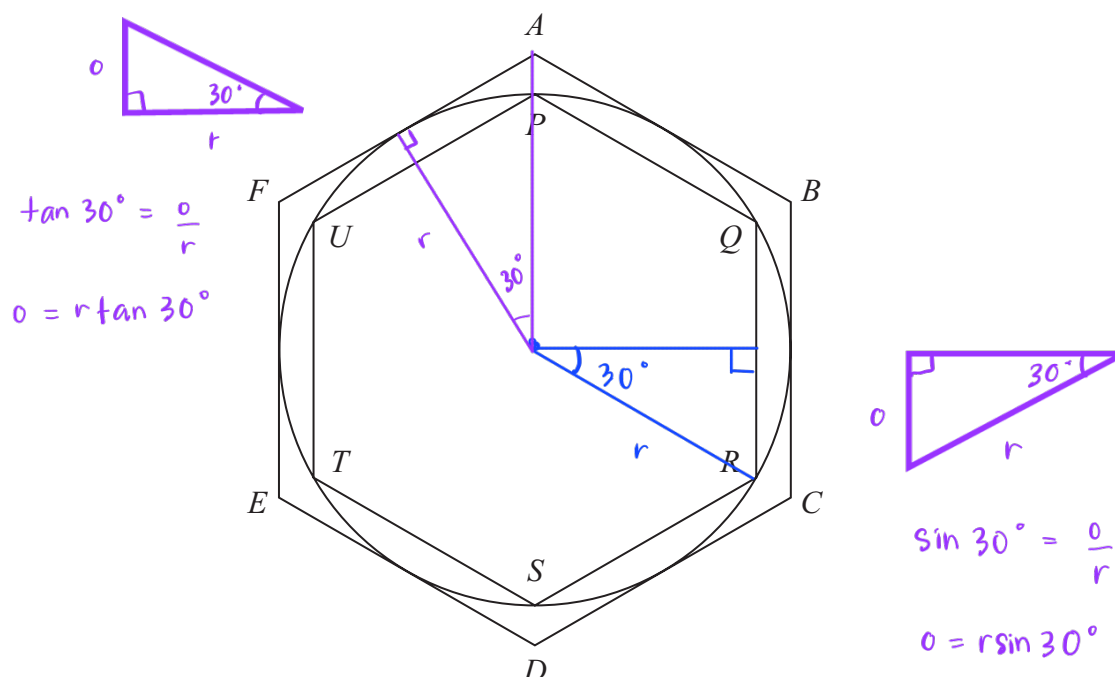


1 The diagram shows a circle, radius r cm and two regular hexagons.



Each side of the larger hexagon $ABCDEF$ is a tangent to the circle.

Each side of the smaller hexagon $PQRSTU$ is a chord of the circle.

By considering perimeters, show that

$$3 < \pi < 2\sqrt{3}$$

$$r \sin 30^\circ = \frac{1}{2} QR$$

$$r \left(\frac{1}{2} \right) = \frac{1}{2} QR$$

$$r = QR$$

$$\text{Perimeter small hexagon} = 6r \quad (1)$$

$$r \tan 30^\circ = \frac{1}{2} AF$$

$$r \left(\frac{\sqrt{3}}{3} \right) = \frac{1}{2} AF$$

$$AF = \frac{2r\sqrt{3}}{3}$$

$$\text{Perimeter large hexagon} = \frac{2r\sqrt{3}}{3} \times 6$$

$$= 4\sqrt{3}r \quad (1)$$

Perimeter
Small
hexagon

< Perimeter
circle

< Perimeter
large
hexagon

$$\begin{aligned} 6r &< 2\pi r < 4\sqrt{3}r \quad (1) \\ \div r &\downarrow 6 < 2\pi < 4\sqrt{3} \quad \div r \\ \div 2 &\downarrow 3 < \pi < 2\sqrt{3} \quad \div 2 \end{aligned}$$

(1)

$$3 < \pi < 2\sqrt{3} \quad (\text{shown})$$

(Total for Question 1 is 4 marks)